Math 55 Quiz 5 DIS 105

Name: ______________________________ 7 Mar 2022

1. Let f_n be the n^{th} Fibonacci number. (Recall that these are defined by $f_1 = 1, f_2 = 1$, and $f_{n+2} = f_n + f_{n+1}$ for $n \geq 1$.)

   Use induction to show that $f_2 + f_4 + f_6 + ... + f_{2n} = f_{2n+1} - 1$ for every $n \geq 1$. [5 points]

   Let $P(n)$ be the proposition that $f_2 + f_4 + f_6 + ... + f_{2n} = f_{2n+1} - 1$.
   For $n = 1$, $f_2 = 1 = f_3 - 1$, so $P(1)$ is true.
   Suppose $P(n)$ is true for some $n \geq 1$.
   Then

$$\begin{aligned} f_2 + f_4 + f_6 + ... + f_{2n+2} &= f_2 + f_4 + f_6 + ... + f_{2n} + f_{2n+2} \\ &= f_{2n+1} - 1 + f_{2n+2} \\ &= (f_{2n+1} + f_{2n+2}) - 1 \\ &= f_{2n+3} - 1 \end{aligned}$$

   So $P(n+1)$ is true.
   By mathematical induction, $P(n)$ is true for every $n \geq 1$.

2. Find the number of bit strings of length ten that begin with 101 or end with 010. Explain your answer. [5 points]

There are 2^7 bit strings of length ten that begin with 101, since each of the final 7 digits can be either 0 or 1. By the same reasoning, there are 2^7 bit strings of length ten that end with 010.
Also, similarly, there are 2^4 bit strings of length ten that begin with 101 and end with 010. So the number of bit strings of length ten that begin with 101 or end with 010 is $2^7+2^7-2^4 = 240$.